

Non-Gaussian signatures of Tachyacoustic Cosmology

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I investigate non-Gaussian signatures in the context of tachyacoustic cosmology, that is, a noninflationary model with superluminal speed of sound. I calculate the full non-Gaussian amplitude \mathcal{A} , its size f_{NL} , and corresponding shapes for a red-tilted spectrum of primordial scalar perturbations. Specifically, for cusciton-like models I show that $f_{\text{NL}} \sim \mathcal{O}(1)$, and the shape of its non-Gaussian amplitude peaks for both equilateral and local configurations, the latter being dominant. These results, albeit similar, are quantitatively distinct from the corresponding ones obtained by Magueijo *et. al* in the context of superluminal bimetric models.

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I. INTRODUCTION

The physics of the very early universe is a rich investigation field, for there is a plethora of potential models that solve, at least partially, the well-known problems of the standard cosmological paradigm. Inflationary cosmology [1] is surely the most successful model, but alternatives to it have been proposed in the recent past: pre-big bang cosmology [2], ekpyrotic and cyclic models [3], nonsingular quantum cosmological models [4], noncanonical models [5, 6], among others. All of them lead to a near-scale invariant spectrum, so that a very important question emerges: how can we falsify such distinct cosmologies? An answer can be provided by the investigation of their non-Gaussian signatures; single-field inflation, for example, predicts a nearly-Gaussian CMB anisotropy field, whereas some noncanonical models, for example, can predict large deviations from Gaussianity, as in DBI inflation (see, for example, references [7] for large-field polynomial models, and [8]). Therefore, the study of non-Gaussian signatures can be a very important tool to rule out many models proposed to cope with the problems of the very early universe.

In particular, noncanonical Lagrangians lead to models with varying speed of sound, a feature which substantially enhances or diminishes their non-Gaussian amplitudes, since they are usually related to terms with a c_s^{-2} dependence [9–11]; hence, for $c_s < 1$, such terms dominate, enhancing non-Gaussianities as in the case of DBI inflation. For superluminal models, $c_s \gg 1$, such terms become subdominant, so that the size of the non-Gaussian amplitude becomes $f_{\text{NL}} \sim 1$ [12, 13], which distinguishes this class of superluminal models from inflation (for which $f_{\text{NL}} \sim \mathcal{O}(0.01)$) [14, 15]. In [12], the authors take into account a minimal and a nonminimal bimetric model, that is, models in which a disformal transformation between matter and gravity metric is evoked, and has the form $\hat{g}_{\mu\nu} = g_{\mu\nu} - B\partial_\mu\phi\partial_\nu\phi$, where the coupling B is regarded as a constant in the former, whereas in the latter it can run with ϕ . They then calculate non-Gaussian amplitudes for the nonminimal bimetric model in the superluminal limit, showing that it only mildly depends on the tilt $n_s - 1$ of the power spectrum. In [13], the authors take a step further and consider non-Gaussianity for a wider class of models without slow-roll and exact scale invariance assumptions, including DBI and bimetric models, and study the superluminal limit of the latter. However, despite the generality of their analysis, there was still room for another class of superluminal models, which we presented in [6] and dubbed *tachyacoustic*.

Tachyacoustic cosmology is a noncanonical, noninflationary model with superluminal speed of sound, in which a nearly scale-invariant spectrum is generated by quantum perturbations redshifted outside of a shrinking acoustic horizon. Since this model is noninflationary, and the scale factor is a power law of time, $a(t) \propto t^{1/\epsilon}$, where $\epsilon = \text{const}$ is the flow parameter associated with the ratio $-\dot{H}/H$, we can take a radiative equation of state for the model, discarding the need of a reheating period to make the scalar field decay into radiation. Also, as shown recently in [16], DBI and cusciton-like cosmological solutions also exhibit attractor behavior. Having this model a nearly scale invariant spectrum of perturbations, as mentioned, the next step to take is exactly the analysis of its non-Gaussian features, which is the subject of this paper.

As we shall see, the distinguishing feature of the tachyacoustic model with its *cusciton-like* Lagrangian¹ with regard

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¹ A *cusciton* is a causal field with infinite speed of sound, originally proposed in [17]. From the flow hierarchy devised for arbitrary

to the other superluminal models explored in [12, 13] concerns the contribution of the term $\dot{\zeta}^3$ in the cubic action [9, 10] to the full non-Gaussian amplitude, giving rise to an extra term with a linear dependence on the flow parameters ϵ and s . This dependence is not present either in DBI models (since the coefficient of $\dot{\zeta}^3$ in the third-order action is identically null in this case), or in disformal bimetric model, since by projecting its scalar-field action in the Einstein frame one obtains a DBI-like action. Therefore, the tachyacoustic model “inherits” an extra contribution from the $\dot{\zeta}^3$ term, providing a different value for f_{NL} compared to the other superluminal models. Despite small, such difference might be observable in the near future, what is of great importance for falsifying superluminal noncanonical models.

This paper is organized as follows. In Section II A I review the basics of tachyacoustic cosmology, and in Section II B I derive some important results concerning the power spectrum. In Section III A I review the third-order action in the fluctuations, whereas in III B I calculate the $\dot{\zeta}^3$ contribution to the non-Gaussian amplitudes, which is absent in other models. In Sections III C and III D I discuss, respectively, the size and the shapes of the non-Gaussian amplitudes for the cuscuton-like tachyacoustic model. In IV I summarize the main results of this paper.

II. TACHYACOUSTIC COSMOLOGY

A. Basics

One of the crucial problems of the very-early cosmology is the presence of a horizon that prevents a causal explanation for the observed features of the universe. The inflationary paradigm tackles this problem by means of a huge accelerated expansion in the very early universe, enforcing physical scales to be deep inside the horizon and, in consequence, in causal contact. However, if light could have travelled much faster in the early universe, all scales would have been naturally in causal contact, so that the horizon problem would not be an issue at all. This is the basic proposal of the varying speed of light theories (VSL) [18]. Despite VSL theories fail to explain cosmic structures, their essential idea remains a very insightful approach to search for alternatives to the inflationary paradigm.

Along with the speed of light, another velocity parameter comes into play when studying cosmological perturbations: the speed of sound parameter c_s . In the standard inflationary scenario, c_s is a constant and coincides with the speed of light. However, in k-essence models, the presence of a noncanonical kinetic term gives rise to the possibility of a varying speed of sound. Also, such models possess two horizons, a Hubble horizon and an acoustic horizon, which have independent dynamics; since the acoustic horizon depend on the speed of sound, it can be large or even infinite at early times for large values of c_s , solving the cosmological horizon problem without inflation. In this case, as in VSL models, superluminal propagation is again evoked, for c_s represents the speed of propagation of the perturbations on a nontrivial background; then, a relevant question now arises: are these models causally consistent?

Fortunately, causal problems are usually model-dependent. As pointed out in references [19] and [20], causal problems do not affect general k-essence models or bimetric theories; also, cuscuton models are free from them [17]. In particular, it can be shown that the noncanonical structure of a general k-essence model induces an “acoustic” metric $G^{\mu\nu}$ [20]; hence, this two-metric model possess two distinct local causal structures, the first one being the usual light cone (connected with the background metric $g_{\mu\nu}$ of the manifold \mathcal{M}), and the second one connected to the speed of sound (thereby the “acoustic cone”). It can be shown that if there exists a time coordinate t with respect to the background metric (which is everywhere future directed) which plays a role of global time for *both* spacetimes $(\mathcal{M}, g_{\mu\nu})$ and $(\mathcal{M}, G_{\mu\nu}^{-1})$, which guarantees the absence of closed causal curves for k-essence models built from homogeneous scalar fields on a FRW background [20]. Since the causal structure for small perturbations is determined by the acoustic cone, we can study superluminal propagation safely, for no causal paradoxes arise.

Tachyacoustic cosmology is a particular solution of a wider class of k-essence models (see Bean *et al.* [21]), which we briefly review below. Consider a general Lagrangian of the form $\mathcal{L} = \mathcal{L}[X, \phi]$, where $2X = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ is the canonical kinetic term ($X > 0$ according to our choice of the metric signature). The energy density ρ and pressure p are given by

$$P = \mathcal{L}(X, \phi), \quad (1)$$

$$\rho = 2X\mathcal{L}_X - \mathcal{L}, \quad (2)$$

k-essence models [21] we reconstructed an action similar to the one associated with the cuscuton plus a canonical kinetic term, which accounts for the “cuscuton-like” term we proposed in [6].

whereas the speed of sound is given by

$$c_s^2 \equiv \frac{P_X}{\rho_X} = \left(1 + 2X \frac{\mathcal{L}_{XX}}{\mathcal{L}_X}\right)^{-1}, \quad (3)$$

where the subscript “X” indicates a derivative with respect to the kinetic term. The Friedmann equation can be written in terms of the reduced Planck mass $M_P = 1/\sqrt{8\pi G}$

$$H^2 = \frac{1}{3M_P^2} \rho = \frac{1}{3M_P^2} (2X\mathcal{L}_X - \mathcal{L}). \quad (4)$$

For monotonic field evolution, the field value ϕ can be used as a “clock”, and all other quantities expressed as functions of ϕ , for example $X = X(\phi)$, $\mathcal{L} = \mathcal{L}[X(\phi), \phi]$, and so on. We consider the homogeneous case, so that $\dot{\phi} = \sqrt{2X}$. Next, using

$$\frac{d}{dt} = \dot{\phi} \frac{d}{d\phi} = \sqrt{2X} \frac{d}{d\phi}, \quad (5)$$

we can re-write the Friedmann equation as the *Hamilton Jacobi* equation

$$\dot{\phi} = \sqrt{2X} = -\frac{2M_P^2}{\mathcal{L}_X} H'(\phi), \quad (6)$$

where a prime denotes a derivative with respect to the field ϕ . The number of e-folds dN can similarly be re-written in terms of $d\phi$ by:

$$dN \equiv -H dt = -\frac{H}{\sqrt{2X}} d\phi = \frac{\mathcal{L}_X}{2M_P^2} \left(\frac{H(\phi)}{H'(\phi)} \right) d\phi. \quad (7)$$

We introduce a hierarchy of flow parameters, the first three defined by

$$\epsilon(\phi) \equiv -\frac{\dot{H}}{H^2} = \frac{2M_P^2}{\mathcal{L}_X} \left(\frac{H'(\phi)}{H(\phi)} \right)^2, \quad (8)$$

$$s(\phi) \equiv \frac{\dot{c}_s}{c_s H} = -\frac{2M_P^2}{\mathcal{L}_X} \frac{H'(\phi)}{H(\phi)} \frac{c'_s(\phi)}{c_s(\phi)}, \quad (9)$$

$$\tilde{s}(\phi) \equiv -\frac{\dot{\mathcal{L}}_X}{H \mathcal{L}_X} = \frac{2M_P^2}{\mathcal{L}_X} \frac{H'(\phi)}{H(\phi)} \frac{\mathcal{L}'_X}{\mathcal{L}_X}, \quad (10)$$

where we have used equation (6). Note that using definitions (4), (8), we can write down the equation of state for a k-essence fluid as

$$P = \left(\frac{2}{3}\epsilon - 1 \right) \rho, \quad (11)$$

where we have used equations (1) and (2). Therefore, the equation of state parameter w can be written in terms of ϵ and, conversely,

$$\epsilon = \frac{3}{2} (1 + w). \quad (12)$$

Hence, a matter-dominated ($w = 0$) tachyacoustic model has $\epsilon = 3/2$, whereas a radiation dominated one ($w = 1/3$) has $\epsilon = 2$.

In the case of Tachyacoustic Cosmology, we take all flow parameters to be constant, and then solve the flow hierarchy, reconstructing the action at the end. In particular, we find that the Hubble parameter and the speed of sound evolve as

$$H(\phi) = H_0 \left(\frac{\phi}{\phi_0} \right)^{\epsilon/s}, \quad (13)$$

$$c_s(\phi) = \frac{\phi_0}{\phi}, \quad (14)$$

and taking $\tilde{s} = -2s$, the resulting Lagrangian has a cuscuton-like form [6]

$$\mathcal{L}(X, \phi) = 2f(\phi)\sqrt{X} + CX - V(\phi), \quad (15)$$

where $f(\phi)$ is the analog of the inverse brane tension, $V(\phi)$ is the potential and C is a constant given by

$$C = \frac{2M_P^2\epsilon}{s^2\phi_0^2}. \quad (16)$$

To conclude this review, is important to mention that, as pointed out at the beginning of this section and demonstrated in [6], tachyacoustic cosmology is free from causal paradoxes. Also, as the acoustic horizon is given by

$$D_H \simeq \frac{c_S}{aH}, \quad (17)$$

we can be shown that its conformal-time dependency goes like

$$D_H \propto \frac{c_S}{aH} \propto e^{(1-\epsilon-s)N} \propto \tau^{(1-\epsilon-s)/(1-\epsilon)}. \quad (18)$$

Therefore the condition for a shrinking acoustic horizon, $1 - \epsilon - s > 0$, is *not* identical to accelerated expansion. For $\epsilon > 1$ and $s < 1 - \epsilon$, the expansion is non-inflationary, the Hubble horizon is growing in comoving units, and the acoustic horizon is shrinking. The initial singularity is at $\tau = 0$, and we see immediately that for the tachyacoustic solution, the speed of sound in the scalar field is *infinite* at the initial singularity, and the acoustic horizon is likewise infinite in size. Therefore, such a cosmology presents no “horizon problem” in the usual sense, since even a spatially infinite spacetime is causally connected on the initial-time boundary.

There are two other important parameters to be taken into account in our discussion. They appear as the coefficient of the term ζ^3 when one expands a general noncanonical action up to third-order in the fluctuations, in the following combination:

$$\Lambda \equiv \Sigma \left(1 - \frac{1}{c_s^2}\right) + 2\lambda, \quad (19)$$

where the parameters Σ and λ are given by [9, 10],

$$\Sigma = X\mathcal{L}_X + 2X^2\mathcal{L}_{XX} = \frac{H^2\epsilon}{c_s^2}, \quad (20)$$

$$\lambda = X^2\mathcal{L}_{XX} + \frac{2}{3}X^3\mathcal{L}_{XXX}. \quad (21)$$

The parameter λ can also be written as [9, 11]

$$\lambda = \frac{\Sigma}{6} \left(\frac{2f_X + 1}{c_s^2} - 1 \right), \quad (22)$$

where f_X and the “kinetic part” ϵ_X of the flow parameter $\epsilon = \epsilon_X + \epsilon_\phi$, are respectively given by

$$f_X = \frac{\epsilon s}{3\epsilon_X}, \quad \epsilon_X = -\frac{\dot{X}}{H^2} \frac{\partial H}{\partial X}. \quad (23)$$

The Σ term as presented in the right-hand side of equation (20) holds the same form regardless of the noncanonical model involved, but the λ term may vary according to the underlying Lagrangian. For DBI-like Lagrangians, for instance, λ is given exactly by [10]

$$\lambda_{DBI} = \frac{H^2\epsilon}{2c_s^4} (1 - c_s^2), \quad (24)$$

whereas for an arbitrary noncanonical model expression (22) is generally employed with the assumption that f_X is constant [11]. Again, for DBI-like models, from (19), (20) and (24) it is straightforward to show that

$$\Lambda_{DBI} = 0, \quad (25)$$

so that the $\dot{\zeta}^3$ contribution to the action vanishes identically. This fact accounts for the absence of the $\mathcal{A}_{\dot{\zeta}^3}$ contribution to the three-point function amplitude of DBI models. The same happens to disformal bimetric models, as argued in the Introduction, for their action reduce to a DBI-like one when projected in the Einstein frame.

In the cusciton-like Lagrangian (15), we do not have to worry about λ , for it is identically null:

$$\lambda_{cusc} = 0, \quad (26)$$

so that

$$\Lambda_{cusc} = \frac{H^2 \epsilon}{c_s^2} \left(1 - \frac{1}{c_s^2} \right); \quad (27)$$

hence, the coefficient of the $\dot{\zeta}^3$ -term does not vanish, unlike the superluminal models described above, yielding a $\mathcal{A}_{\dot{\zeta}^3}$ contribution to the amplitude.

B. Perturbations

In [6] we have solved the mode equation for an arbitrary noncanonical model [22] in the case of constant flow parameters (as defined in equations (8-10)). In this paper I will take a slightly different path, using the so-called “sound-horizon” time $dy = c_s d\tau$ instead of the conformal time τ [11], just for the sake of comparisons with the results obtained in [13].

Taking ϵ , s , \bar{s} constant, using definitions (8,9), and defining

$$\gamma \equiv \epsilon + s - 1, \quad (28)$$

it follows that

$$y = \frac{c_s}{\gamma a H}, \quad (29)$$

and

$$a \sim (-y)^{1/\gamma}, \quad c_s \sim (-y)^{s/\gamma}, \quad H \sim (-y)^{-\epsilon/\gamma}. \quad (30)$$

We next turn to the second-order action for the curvature perturbation ζ [22]:

$$S_2 = \frac{M_P^2}{2} \int d^3x d\tau \, z^2 \left[\left(\frac{d\zeta}{d\tau} \right)^2 - c_s^2 (\nabla \zeta)^2 \right], \quad (31)$$

where z is defined as $z = a\sqrt{2\epsilon}/c_s$. From now on, a prime $'$ indicates a derivative with respect to y ; then, defining

$$q \equiv \sqrt{c_s} z, \quad (32)$$

action (31) becomes

$$S_2 = \frac{M_P^2}{2} \int d^3x dy \, q^2 [\zeta'^2 - (\nabla \zeta)^2]; \quad (33)$$

next, introducing the canonical variable $v = M_P q \zeta$, from (33) we derive the mode equation

$$v_k'' + \left(k^2 - \frac{q''}{q} \right) v_k = 0. \quad (34)$$

After a little algebra we can rewrite the last term in the equation above as

$$\frac{q''}{q} = \frac{1}{y^2} \left(\nu^2 - \frac{1}{4} \right), \quad (35)$$

where

$$\nu \equiv \frac{\epsilon + 2s - 3}{2(\epsilon + s - 1)}. \quad (36)$$

The scale-invariant limit is achieved when $q''/q = 2/y^2$, so that $\nu = 3/2$ in this case. Since curvature perturbations ζ are quantum fields, we expand them in terms of creation and annihilation operators,

$$\zeta(y, \mathbf{k}) = u_k(y)a_{\mathbf{k}} + u_k^*(y)a_{-\mathbf{k}}^\dagger, \quad (37)$$

so that the solution for $v_k(y)$ associated with the Bunch Davis vacuum is

$$v_k(y) = \frac{\sqrt{\pi}}{2} \sqrt{-y} H_\nu^{(1)}(-ky), \quad (38)$$

where $H_\nu^{(1)}$ are Hankel functions of the first kind. Next, from the modes introduced in (37) and solution (38) we find that

$$u_k(y) = \frac{c_s^{1/2}}{aM_P 2^{3/2}} \sqrt{\frac{\pi}{\epsilon}} \sqrt{-y} H_\nu^{(1)}(-ky) \approx -\frac{iH\gamma}{2M_P \sqrt{c_s k^3 \epsilon}} \left(\frac{-ky}{2}\right)^{3/2-\nu} (1 + iky) e^{-iky}, \quad (39)$$

where we have used the following expansion of Hankel functions for small arguments, $|ky| \ll 1$,

$$H_\nu^{(1)}(-ky) = -i \frac{2^\nu \Gamma(\nu) (-ky)^{-\nu}}{\pi} [1 + iky + \mathcal{O}(ky)^2] e^{-iky}, \quad (40)$$

together with the assumption that $\Gamma(\nu \approx 3/2) \approx \sqrt{\pi}/2$, which corresponds to nearly scale invariance, as desired. From expression (40) we can also compute the derivative of $u_k(y)$ with respect to y ,

$$u'_k(y) \approx -i \frac{H(\epsilon + s - 1)}{2M_P \sqrt{c_s k^3 \epsilon}} \left(\frac{-ky}{2}\right)^{3/2-\nu} k^2 y e^{-iky}, \quad (41)$$

which plays an important role in the computations of section III B.

From the definition of the ζ power spectrum

$$P_\zeta \equiv \frac{1}{2\pi^2} k^3 |\zeta_k|^2, \quad (42)$$

we find,

$$P_\zeta = \frac{(\epsilon + s - 1)^2 2^{2\nu-3} \bar{H}^2}{2(2\pi)^2 M_P^2 \epsilon \bar{c}_s}, \quad (43)$$

where the bar refers to quantities evaluated at sound horizon-crossing, $y = k^{-1}$. From the computation of the power spectrum we find the expression for the scalar spectral index n_s , which is related to ν and the flow parameters ϵ and s by

$$n_s - 1 = 3 - 2\nu = \frac{2\epsilon + s}{s + \epsilon - 1}. \quad (44)$$

III. NON-GAUSSIAN SIGNATURES

A. Third-order action

In order to investigate non-Gaussian signatures, we basically compute the three-point function for the curvature perturbations, starting from the third-order action in the ADM formalism [9, 10]:

$$\begin{aligned} S_3 = & M_P^2 \int dt d^3x \left\{ -a^3 \left[\Sigma \left(1 - \frac{1}{c_s^2} \right) + 2\lambda \right] \frac{\dot{\zeta}^3}{H^3} + \frac{a^3 \epsilon}{c_s^4} (\epsilon - 3 + 3c_s^2) \zeta \dot{\zeta}^2 - 2a \frac{\epsilon}{c_s^2} \dot{\zeta} (\partial \zeta) (\partial \chi) \right. \\ & + \frac{a\epsilon}{c_s^2} (\epsilon - 2s + 1 - c_s^2) \zeta (\partial \zeta)^2 + \frac{a^3 \epsilon}{2c_s^2} \frac{d}{dt} \left(\frac{\eta}{c_s^2} \right) \zeta^2 \dot{\zeta} + \frac{\epsilon}{2a} (\partial \zeta) (\partial \chi) \partial^2 \chi + \frac{\epsilon}{4a} (\partial^2 \zeta) (\partial \chi)^2 \\ & \left. + 2f(\zeta) \frac{\delta L}{\delta \zeta} \Big|_1 \right\}; \end{aligned} \quad (45)$$

such expression is valid outside of the slow-roll approximation and for any time-independent sound speed. In (45) the dots denote derivatives with respect to proper time t , and χ is defined as

$$\partial^2 \chi = \frac{a^2 \epsilon}{c_s^2} \dot{\zeta}, \quad (46)$$

whereas the elements appearing in the last term of the action (45) are

$$f(\zeta) = \frac{\eta}{4c_s^2} \zeta^2 + \frac{1}{c_s^2 H} \zeta \dot{\zeta} + \frac{1}{4a^2 H^2} [-(\partial \zeta)(\partial \zeta) + \partial^{-2}(\partial_i \partial_j (\partial_i \zeta \partial_j \zeta))] + \frac{1}{2a^2 H} [(\partial \zeta)(\partial \chi) - \partial^{-2}(\partial_i \partial_j (\partial_i \zeta \partial_j \chi))], \quad (47)$$

$$\left. \frac{\delta L}{\delta \zeta} \right|_1 = a \left(\frac{d\partial^2 \chi}{dt} + H \partial^2 \chi - \epsilon \partial^2 \zeta \right), \quad (48)$$

where ∂^{-2} is the inverse Laplacian. Note that (48) corresponds to the linearized equations of motion, and can be absorbed by a field redefinition

$$\zeta \rightarrow \zeta_n + f(\zeta_n). \quad (49)$$

Since in the limit $y \rightarrow 0$ one has

$$\frac{H}{c_s^{1/2}} \sim (-y)^{\nu-3/2}, \quad (50)$$

it is clear from expression (39) that $\zeta \rightarrow \text{const}$ on superhorizon scales, so that we can drop the second term in (47), and we are left with the field redefinition $\zeta \rightarrow \zeta_n + \eta \zeta_n^2 / (4c_s^2)$; in the present case, for constant flow parameters we get $\eta = 0$, so that we can simply skip the last term in the action (45).

B. Amplitudes

I next turn to the calculation of the three-point function which leads to the f_{NL} parameter, following the same steps as taken in [11]. In the interaction picture, the three-point function is given by

$$\langle \zeta(t, \mathbf{k}_1) \zeta(t, \mathbf{k}_2) \zeta(t, \mathbf{k}_3) \rangle = -i \int_{t_0}^t dt' \langle [\zeta(t, \mathbf{k}_1) \zeta(t, \mathbf{k}_2) \zeta(t, \mathbf{k}_3), H_{\text{int}}(t')] \rangle, \quad (51)$$

where $H_{\text{int}} = -L_{\text{int}}$ is the interaction Hamiltonian derived from the corresponding Lagrangian (45), and vacuum expectation values are evaluated with respect to the interacting vacuum $|\Omega\rangle$ (a thorough discussion on this point can be found in [9]); t_0 is some sufficiently early time. Except from the $\dot{\zeta}^3$ term, as discussed in the Introduction, all the amplitudes derived have the same form as the amplitudes evaluated in [13]. Then, I shall evaluate here only the \mathcal{A}_{ζ^3} contribution to the non-Gaussian amplitude, which is derived from

$$S_{\zeta^3} = M_P^2 \int_{-\infty+i\epsilon}^{y_{\text{end}}} dy d^3x \frac{c_s^3}{H^3} \Sigma \left(1 - \frac{1}{c_s^2} \right) \zeta'^3, \quad (52)$$

where we have changed the time variable t with the sound-horizon time variable $dy = c_s dt/a$, and used (26). The parameter $y_{\text{end}} < 0$ indicates the “end” of the deceleration period with infinite speed of sound (in [11], it refers to the end of inflationary or ekpyrotic phase). Next, substituting contribution (37) and (52) into (51), and using the commutation relations $[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')$, we find

$$\begin{aligned} \langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle_{\zeta^3} &= -(2\pi)^3 i \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) u_{k_1}(y_{\text{end}}) u_{k_2}(y_{\text{end}}) u_{k_3}(y_{\text{end}}) \\ &\times \left\{ \int_{-\infty+i\epsilon}^{y_{\text{end}}} dy \frac{ac_s^2}{H^3} \Sigma \left(1 - \frac{1}{c_s^2} \right) u_{k_1}^*(y)' u_{k_2}^*(y)' u_{k_3}^*(y)' + \text{perm.} + \text{c.c.} \right\}. \end{aligned} \quad (53)$$

Also, substituting (20), (29), (39) and (41) into equation (53), we find

$$\begin{aligned} \langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle_{\zeta^3} &= -(2\pi)^3 i \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{\gamma^6}{2^{15-6\nu} M_P^4 \epsilon^3} \frac{H_{\text{end}}^3}{c_{s,\text{end}}^{3/2}} (k_1 k_2 k_3)^{2-2\nu} |y_{\text{end}}|^{9/2-3\nu} \\ &\times \frac{\epsilon}{\gamma} \int_{-\infty+i\epsilon}^{y_{\text{end}}} dy \frac{H}{c_s^{1/2}} \left(1 - \frac{1}{c_s^2} \right) y^2 y^{9/2-3\nu} e^{iKy} + \text{perm.} + \text{c.c.}, \end{aligned} \quad (54)$$

where $K \equiv k_1 + k_2 + k_3$. The integral appearing in the expression can be simplified as follows: first, from (30) we get the following expressions,

$$\frac{H}{c_s^{1/2}} = \frac{H_{\text{end}}}{c_{s,\text{end}}^{1/2}} \left(\frac{y}{y_{\text{end}}} \right)^\alpha, \quad \frac{H}{c_s^{5/2}} \left(\frac{y}{y_{\text{end}}} \right)^{\nu-3/2} = \frac{H_{\text{end}}}{c_{s,\text{end}}^{5/2}} \left(\frac{y}{y_{\text{end}}} \right)^\beta, \quad (55)$$

where we have defined the exponents

$$\alpha \equiv 3 - 2\nu = \frac{2\epsilon + s}{\epsilon + s - 1}, \quad \beta \equiv \frac{2\epsilon - s}{\epsilon + s - 1}; \quad (56)$$

next, defining

$$\mathcal{I}_n(p) \equiv \text{Im} \left\{ \int_{-\infty+i\epsilon}^{y_{\text{end}}} dy \left(\frac{y}{y_{\text{end}}} \right)^p (-iy)^n e^{iKy} \right\}, \quad (57)$$

we can reduce expression (54) to

$$\begin{aligned} \langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle_{\zeta^3} &= (2\pi)^3 i \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) (k_1 k_2 k_3)^{2-2\nu} \frac{\gamma^5}{2^{15-6\nu} M_P^4 \epsilon^2} \frac{H_{\text{end}}^4}{c_{s,\text{end}}^2} |y_{\text{end}}|^{9-6\nu} \\ &\quad \times \left[\frac{1}{c_{s,\text{end}}^2} \mathcal{I}_2(\beta) - \mathcal{I}_2(\alpha) \right] + \text{perm.} + \text{c.c.} \end{aligned} \quad (58)$$

It is important to stress that we took the imaginary part of integral (57) for this is the only part relevant for this calculation [11]. For all modes of interest, $K|y_{\text{end}}|$ is a small quantity; then, as discussed in [15], the above choice of integration contour picks up the appropriate interacting “in-in” vacuum at $|y| \rightarrow \infty$ and takes care of the oscillating behavior of the exponential.

We can further simplify expression (58) by relating the quantities evaluated at $y = y_{\text{end}}$ to their corresponding values at sound horizon-crossing, $y = K^{-1}$,

$$H_{\text{end}} = \bar{H} (K y_{\text{end}})^{-\epsilon/\gamma}, \quad c_{s,\text{end}} = \bar{c}_s (K y_{\text{end}})^{s/\gamma}, \quad (59)$$

so that we use the expression for the ζ power-spectrum (43) to absorb the constants \bar{H} and \bar{c}_s in (58); the final result is

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle_{\zeta^3} = (2\pi)^7 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta^2 \frac{1}{\prod_j k_j^3} \mathcal{A}_{\zeta^3}, \quad (60)$$

where

$$\mathcal{A}_{\zeta^3} = -\frac{3}{4} \frac{\epsilon + s - 1}{\bar{c}_s^2} \left(\frac{k_1 k_2 k_3}{2K^3} \right)^{n_s-1} \left[I_{\zeta^3}(\beta) - \bar{c}_s^2 I_{\zeta^3}(\alpha) \right], \quad (61)$$

and

$$I_{\zeta^3}(p) \equiv -(k_1 k_2 k_3)^2 (K |y_{\text{end}}|)^p \mathcal{I}_2(p). \quad (62)$$

Using the same methods the other relevant integrals are given by (they coincide with the ones computed in [13]):

$$\mathcal{A}_{\zeta^2} = \frac{1}{4\bar{c}_s^2} \left(\frac{k_1 k_2 k_3}{2K^3} \right)^{n_s-1} \left[(\epsilon - 3) I_{\zeta^2}(\beta) + 3\bar{c}_s^2 I_{\zeta^2}(\alpha) \right], \quad (63)$$

$$\mathcal{A}_{\zeta(\partial\zeta)^2} = \frac{1}{8\bar{c}_s^2} \left(\frac{k_1 k_2 k_3}{2K^3} \right)^{n_s-1} \left[(\epsilon - 2s + 1) I_{\zeta(\partial\zeta)^2}(\beta) - \bar{c}_s^2 I_{\zeta(\partial\zeta)^2}(\alpha) \right], \quad (64)$$

where

$$I_{\zeta^2}(p) \equiv -(K |y_{\text{end}}|)^p \left\{ [\mathcal{I}_0(p) + K \mathcal{I}_1(p)] \sum_{i < j} k_i^2 k_j^2 - \mathcal{I}_1(p) \frac{1}{K^2} \sum_{i \neq j} k_i^2 k_j^3 \right\}, \quad (65)$$

$$I_{\zeta(\partial\zeta)^2}(\alpha) \equiv (K |y_{\text{end}}|)^p \left(\sum_i k_i^2 \right) \left[\mathcal{I}_{-2}(p) + K \mathcal{I}_{-1}(p) + \mathcal{I}_0(p) \sum_{i < j} k_i k_j + k_1 k_2 k_3 \mathcal{I}_1(p) \right]. \quad (66)$$

Contributions $\mathcal{A}_{\zeta\partial\zeta\partial\chi}$, $\mathcal{A}_{\partial\zeta\partial\chi\partial^2\chi}$ and $\mathcal{A}_{(\partial^2\zeta)(\partial\chi)^2}$ to the full amplitude, as found in [13], are subdominant, for they go to zero as $\bar{c}_s \rightarrow \infty$, so that we neglect them here.

Integrals of the form (57) are convergent for $p + n > -2$ as $y_{\text{end}} \rightarrow 0$ [11], and assume the form

$$\mathcal{I}_n(p) = -(K|y_{\text{end}}|)^{-p} \cos \frac{p\pi}{2} \Gamma(1+p+n) K^{-n-1}, \quad (67)$$

then, in the limit $\bar{c}_s \rightarrow \infty$, the relevant integrals (62), (65) and (66) assume the form

$$I_{\zeta^3}(\alpha) = \frac{(k_1 k_2 k_3)^2}{K^3} \cos \frac{\alpha\pi}{2} \Gamma(3+\alpha), \quad (68)$$

$$I_{\zeta\dot{\zeta}^2}(\alpha) = \cos \frac{\alpha\pi}{2} \Gamma(1+\alpha) \left[\frac{(2+\alpha)}{K} \sum_{i<j} k_i^2 k_j^2 - \frac{(1+\alpha)}{K^2} \sum_{i \neq j} k_i^2 k_j^3 \right], \quad (69)$$

$$I_{\zeta(\partial\zeta)^2}(\alpha) = -\cos \frac{\alpha\pi}{2} \Gamma(1+\alpha) \left(\sum_i k_i^2 \right) \left[\frac{K}{\alpha-1} + \frac{1}{K} \sum_{i<j} k_i k_j + (1+\alpha) \frac{k_1 k_2 k_3}{K} \right], \quad (70)$$

where α is given by (56). Adding up all the relevant contributions to the full amplitude $\mathcal{A} = \mathcal{A}_{\zeta^3} + \mathcal{A}_{\zeta\dot{\zeta}^2} + \mathcal{A}_{\zeta(\partial\zeta)^2}$, given by (61), (63) and (64) respectively, taking the small tilt ($n_s - 1 \ll 1$) and $\bar{c}_s \rightarrow \infty$ limits, we find that the full amplitude for cuscuton-like models is given by

$$\begin{aligned} \mathcal{A}_{\bar{c}_s \rightarrow \infty} = & \left(\frac{k_1 k_2 k_3}{2K^3} \right)^{n_s-1} \left\{ \frac{3}{2} (\epsilon + s - 1) \frac{(k_1 k_2 k_3)^2}{K^3} - \frac{1}{8} \sum_i k_i^3 + \frac{1}{K} \sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2} \sum_{i \neq j} k_i^2 k_j^3 \right. \\ & \left. + (n_s - 1) \left[\frac{9}{4} (\epsilon + s - 1) \frac{(k_1 k_2 k_3)^2}{K^3} - \frac{1}{8} \sum_i k_i^3 - \frac{1}{8} \sum_{i \neq j} k_i k_j^2 + \frac{1}{8} k_1 k_2 k_3 + \frac{1}{2K} \sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2} \sum_{i \neq j} k_i^2 k_j^3 \right] \right\}, \end{aligned} \quad (71)$$

where we have used expressions (68-70) and (73). Note that except for the terms containing an explicit dependence on the flow parameters, $\gamma = \epsilon + s - 1$, equation (28), expression (71) is identical to the one obtained in equation (4.15) of [13]; as pointed out in the Introduction, such contributions are absent in both DBI and nonminimal bimetric models, for they come from the $\dot{\zeta}^3$ contribution in action (45). In the next two sections, I discuss the modifications induced by such contributions with regard to the full superluminal amplitude found in [13].

C. Size of non-Gaussian amplitudes, f_{NL}

The parameter that characterizes the three-point amplitude, f_{NL} , is defined as [14]

$$\zeta = \zeta_g(x) + \frac{3}{5} f_{\text{NL}} \zeta_g^2; \quad (72)$$

following [11], we define f_{NL} at $k_1 = k_2 = k_3 = K/3$, so that

$$f_{\text{NL}} = 30 \frac{\mathcal{A}_{k_1=k_2=k_3}}{K^3}. \quad (73)$$

For the cuscuton-like models, substituting (71) into (73), we get

$$f_{\text{NL}}^c = \left(\frac{1}{54} \right)^{n_s-1} \left\{ \frac{5}{81} (\epsilon + s - 1) + \frac{35}{108} + (n_s - 1) \left[\frac{5}{54} (\epsilon + s - 1) - \frac{25}{27} \right] \right\}, \quad (74)$$

where superscript c stands for *cuscuton*. As for superluminal disformal bimetric theories, the corresponding size of non-Gaussianities f_{NL} is given by equation (3.16) in [13],

$$f_{\text{NL}} \sim 0.28 - 0.04\epsilon - (1.19 + 0.08\epsilon)(n_s - 1); \quad (75)$$

then, for a red-tilted spectrum $n_s \sim 0.96$, we find, for $\epsilon = 0.01$ and $\epsilon = 0.3$ (these are the ϵ values they have chosen to plot Figure 6),

$$f_{\text{NL}} \sim 0.327, \quad f_{\text{NL}} \sim 0.317, \quad (76)$$

respectively; in the case of matter-dominated ($\epsilon = 3/2$) and radiation-dominated ($\epsilon = 2$) cuscuton-like tachyacoustic models one gets

$$f_{\text{NL}}^c \sim 0.266, \quad f_{\text{NL}}^c \sim 0.232 \quad (77)$$

respectively. Then, cuscuton-like models also produce non-Gaussianities of the same order of magnitude as of the other superluminal models, albeit with different magnitudes, as can be seen from the results (76) and (77). Such differences come from the fact that the ζ^3 contribution, present in cuscuton-like models, is not negligible and possesses negative sign (for its γ coefficient, given by equation (28), is *negative* for the flow parameters ϵ and s chosen), so that it *reduces* not only the size of the non-Gaussianities, but their shapes as well.

D. Shapes of non-Gaussian amplitudes

Following the literature [23], I discuss next the shape of the amplitude at fixed K , by means of the dimensionless ratio $\mathcal{A}_{\bar{c}_s \rightarrow \infty}/k_1 k_2 k_3$. It is convenient to change the variables k_2 and k_3 into the dimensionless ones $x_2 = k_2/k_1$ and $x_3 = k_3/k_1$; then, the amplitude to be studied is $\mathcal{A}_{\bar{c}_s \rightarrow \infty}(1, x_2, x_3)/x_2 x_3$ in the $x_2 - x_3$ plane. Without any loss of generality the momenta are ordered such that $x_3 < x_2 < 1$; the triangle inequality implies $x_2 + x_3 > 1$. To avoid plotting the same configuration twice, the amplitude is set to zero outside the range $1 - x_2 < x_3 x_2$.

For the sake of comparison, it is instructive to discuss briefly the results obtained in [12]. First, the full amplitude generated is given by their equation (4.15),

$$\begin{aligned} \mathcal{A}_{\bar{c}_s \rightarrow \infty} = & \left(\frac{k_1 k_2 k_3}{2K^3} \right)^{n_s-1} \left\{ -\frac{1}{8} \sum_i k_i^3 + \frac{1}{K} \sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2} \sum_{i \neq j} k_i^2 k_j^3 \right. \\ & \left. + (n_s - 1) \left[-\frac{1}{8} \sum_i k_i^3 - \frac{1}{8} \sum_{i \neq j} k_i k_j^2 + \frac{1}{8} k_1 k_2 k_3 + \frac{1}{2K} \sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2} \sum_{i \neq j} k_i^2 k_j^3 \right] \right\}, \end{aligned} \quad (78)$$

whose shape is depicted in Figure 1, for the scale-invariant limit, and in Figure 2 for a red-tilted spectrum.

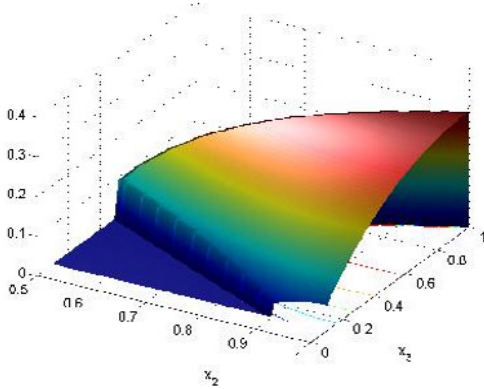


FIG. 1: Non-Gaussian amplitude $\mathcal{A}_{\bar{c}_s \rightarrow \infty}(1, x_2, x_3)/x_2 x_3$ for $n_s = 1$ without the ζ^3 contribution.

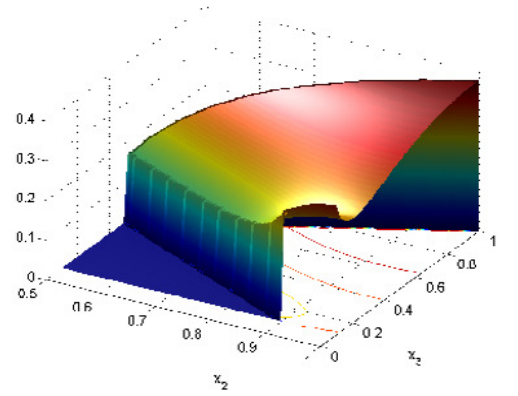


FIG. 2: Non-Gaussian amplitude for the same model, but with $n_s = 0.96$.

As for the cuscuton-like models, we see in Figure 3 that the extra contribution coming from the ζ^3 term modifies the shape dependence. In bimetric models, for instance, the amplitude peaks in the equilateral limit $k_1 = k_2 = k_3$ for both scale-invariant and red-tilted spectra; in the latter, besides, the amplitude also peaks in the local limit $k_3 \ll k_1 = k_2$, which dominates over the equilateral mode. Cuscuton-like models share the same features, as can be seen in the

Figure 3; also, comparing Figures 2 and 3 it is clear that the ζ^3 -terms contributes to reduce the magnitude of the non-Gaussian amplitudes, specially in the equilateral configuration.

This effect is stronger in the radiative cuscuton-like model, which is expected, since the combination (28) yields $\gamma \sim -2.9$ for a radiative equation of state with $\epsilon = 2$ and $s \sim -3.9$, whereas $\gamma \sim -2.4$ for a matter equation of state with $\epsilon = 3/2$ and $s \sim -2.9$. Hence, bimetric and cuscuton-like models, despite their similarities, can be clearly distinguished by means of the different results obtained for their sizes and shapes of non-Gaussianity.

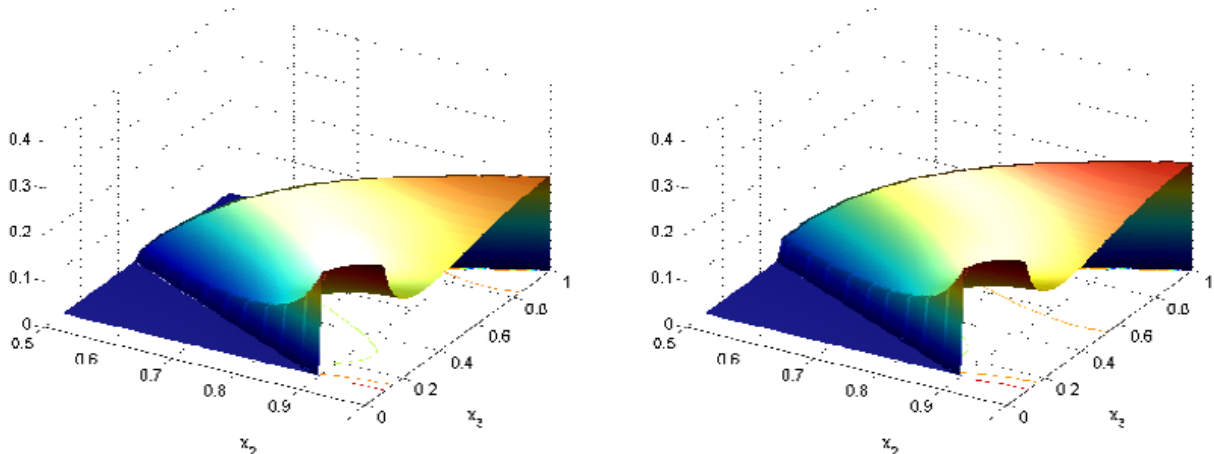


FIG. 3: Non-Gaussian amplitude $\mathcal{A}_{\bar{c}_s \rightarrow \infty}(1, x_2, x_3)/x_2 x_3$ for the cuscuton-like model with $n_s = 0.96$ with a radiative equation of state (left) and with a matter equation of state (right).

IV. CONCLUSIONS

Among the models proposed to solve the puzzles of the very-early universe, those with noncanonical Lagrangians possess an interesting property, the possibility of a varying speed of sound. For subluminal models, as in DBI inflation, a small speed of sound can produce large non-Gaussian amplitudes as $f_{\text{NL}} \sim -100$, for example; in the superluminal case, for instance, as in the nonminimal bimetric model, large non-Gaussianities are suppressed, and the predicted size of non-Gaussianities amounts to be around $f_{\text{NL}} \sim \mathcal{O}(1)$. Such model admits a nearly-scale invariant spectrum, which induces distortions to the equilateral shape of non-Gaussianities, and hence making it distinguishable from other noncanonical subluminal models.

The cuscuton-like models proposed in [6] also possess a superluminal speed of sound, with the advantage of being also noninflationary. This means that, unlike inflationary models, we can choose values greater than one for the flow parameter ϵ , thus eliminating the reheating period for a scalar field with a radiative equation of state. Since they lead to a nearly-scale invariant spectrum, it is natural to take one step further and investigate their non-Gaussian signatures, which was the main subject of this work. After deriving the amplitudes for each term in the third-order action, equation (45), I found that the ζ^3 contribution in cuscuton-like models is nonzero, unlike in the nonminimal bimetric model, due to its DBI-type action. To check whether this contribution induces any significant deviation from the amplitude of bimetric models, I derived an expression for the size of the non-Gaussian amplitudes, given by equation (74), and found that $f_{\text{NL}} \sim \mathcal{O}(1)$, which is of the same order as bimetric models. However, a small difference in the corresponding values of the f_{NL} parameter was found (equations (76) and (77)), due exactly to the negative coefficient γ emerging from the ζ^3 contribution to the three-point amplitude in the cuscuton-like models. Such extra terms reduce the size of non-Gaussianities, as well as the height of their shapes in comparison to bimetric models. Also, the equilateral configuration becomes subdominant, whereas the “squeezed” triangle mode becomes relevant.

Then, cuscuton-like models share similar features with superluminal bimetric models, but they have distinct non-Gaussian signatures, which is evident from the comparison of the shapes of their corresponding amplitudes. Due to the low values predicted for f_{NL} , which are still well below the upcoming constraints to be posed by Planck satellite measurements, it is not yet possible to falsify such models observationally, but only those predicting huge non-Gaussian amplitude sizes.

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